Burlington County Institute of Technology

Medford Campus

Westampton Campus

Algebra II Curriculum

Department: Mathematics
Credits: 5
Revised: August 2023
Board Approval Date: August, 2023

Course Description

Algebra 2 is an advanced course that builds upon the foundations of algebraic concepts introduced in Algebra 1. Through theory, problem-solving, and real-world applications, students will develop critical thinking skills and mathematical fluency.

The course begins with a review of fundamental concepts such as the number system, domain and range, and transformations, and progresses to graphing linear and quadratic functions, working with complex numbers, and solving equations using the quadratic formula.

The subsequent portion of the course delves into trigonometry, exponential and logarithmic functions, polynomials, square root functions, radical expressions, reciprocal functions, and rational functions. Students will explore the properties, graphs, and applications of these functions. They will also learn to solve equations and inequalities involving these functions and apply them to real-world scenarios.

In the final phase of the course, students will focus on statistical analysis, including data distributions and the normal distribution. They will gain skills in analyzing and summarizing data sets, calculating measures of central tendency and variation, and interpreting data in context. Probability concepts will be explored, enabling students to make informed decisions based on statistical data.

Throughout the course, students will engage in problem-solving activities, collaborative discussions, and hands-on exploration of mathematical concepts. Technology tools like graphing calculators and software applications will be used to enhance understanding. By the end of the course, students will have a strong foundation in algebraic principles, sharpened analytical skills, and the ability to apply mathematical concepts in real-world situations.

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Algebra 2 Pacing Guide

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Curriculum Maps

Unit 1: Functions and Relations
Desired Outcomes

NJSLS Mathematics
 	
🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness
 	
Number and Quantity
• The Complex Number System (N-CN)
    🟡 Perform arithmetic operations on complex numbers (N-CN.1. N-CN.2).
    🟡 Use complex numbers in polynomial identities and equations (N-CN.7).
• Quantity (N-Q)
    🔷 Reason quantitatively and use units to solve problems (N-Q.1)
 	
Algebra
• Seeing Structure in Expressions (A-SSE)
    🟩 Interpret the structure of expressions (A-SSE.2).
    🟩 Write expressions in equivalent forms to solve problems (A-SSE.3, A-SSE.4).
• Arithmetic with Polynomials and Rational Expressions (A-APR)
    🟡 Use polynomial identities to solve problems (A-APR.4).
• Creating Equations (A-CED)
    🔷 Create equations that describe numbers or relationships (A-CED.1)
    🟡 Create equations that describe numbers or relationships (A-CED.3).
• Reasoning with Equations and Inequalities (A-REI)
    🔷 Solve equations and inequalities in one variable (A-REI.4b).

🟡 Solve systems of equations (A-REI.6, A-REI.7).

Functions

- Interpreting Functions (F-IF)
  - 🟩 Interpret functions that arise in applications in terms of the context (F-IF.4).
  - 🔷 Analyze functions using different representations (F-IF.7a, F-IF.7.b, F-IF.8a).
- Building Functions (F-BF)
  - 🟡 Build new functions from existing functions (F-BF.3).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	Essential Questions:
○ Evaluating and expanding the number system to include complex and imaginary numbers. ○ Graphs can be examined in detail or in their entirety depending on what information is sought. ○ Graphs provide a means of correlation. ○ Operations can be performed over the complex number system. ○ Symbolic statements can be manipulated to produce other statements of the same relationship. ○ The characteristics of polynomial functions and	○ Why are complex numbers necessary? ○ How are operations and properties of complex numbers related to those of real numbers? ○ Why is math used to model real-world situations? ○ How can you choose a model to represent a set of data? ○ Why are graphs useful?

their representations are useful when solving real world problems. ○ The solutions of polynomial functions can be extended to include the set of complex numbers.	
<u>Students will know:</u> ○ Expressions, equations, and functions are related, but different, algebraic constructs of varied types, such as linear, quadratic, exponential, radical, trigonometric, logarithmic, and many others. ○ Expressions have no equal sign, equations have an equal sign with a constant value on the other side, and functions have an equal sign with a variable on the other side. ○ Expressions can be simplified and evaluated, but not solved or graphed on a coordinate plane. Equations can be simplified and solved, but not graphed. Functions can be graphed on a coordinate plane. ○ Piecewise-defined functions are usually written using two or more algebraic expressions. ○ A common piecewise-linear function, called a step function, consists of a series of line segments that look like steps. ○ The graph of an absolute value function is shaped like a V and Is made up of portions of two lines. ○ The solution set of a linear inequality is the set of all ordered pairs that make the statement true. ○ Absolute value functions and quadratic functions can be graphed using ordered pairs or graphed using the zeros and vertex. ○ Linear, absolute value, and quadratic functions	**<u>Students will be able to:</u>** ○ Identify the mathematical domains and ranges of functions. ○ Write and graph linear functions, quadratic functions, piecewise-defined functions, step functions, absolute value functions and determine reasonable domain and range values. ○ Identify and sketch graphs of parent functions, including linear, absolute value and quadratic functions. Use the parent functions to investigate, describe and predict the effects of changes in a, h, and k in vertex form. ○ Describe transformations of functions and the effect on the domain and range. ○ Interpret and determine the reasonableness of solutions to systems of equations. Use algebraic methods and graphs to solve simple systems of equations in three variables. ○ Use the discriminant to determine the number and type of roots of a quadratic equation. ○ Solve quadratic equations by completing the square. ○ Solve quadratic equations having complex roots using the Quadratic Formula. ○ Use complex numbers to describe the solutions of quadratic equations. Solve quadratic equations using completing the square and the Quadratic

can be transformed and reflected both graphically and algebraically. ○ Piecewise-defined functions, absolute value functions, and quadratic functions can model real-world situations. ○ Quadratic functions can be written in various forms to provide different characteristics of the graph. ○ Quadratic functions can provide real and complex solutions. ○ The critical points provide information about real world phenomena.	Formula with complex solutions. ○ Perform operations with pure imaginary numbers. ○ Perform operations with complex numbers.
Assessment Evidence	
Suggested Performance Tasks: ○ Enrique is recording the number of hamburgers and hot dogs that he eats each week. Hot dogs are 240 calories with 16 grams in fat. Hamburgers are 300 calories with 10 grams in fat. From the hamburgers and hot dogs Enrique eats each week, he decides to have no more than 1,200 total calories and no more than 60 total grams of fat. a. On a coordinate plane, graph the two lines that represent the maximum number of hot dogs and hamburgers Enrique can eat in one week due to the constraints on calories and fat. Graph a "Constraints on Calories" line and a "Constraints on Fat" line and then shade in the desired region. b. Give an example of one point that satisfies the constraints and one that does not	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes

satisfy the constraints. Interpret the meaning in the context of the situation being modeled. Explain your answer. ○ To prepare for a test, three students have been asked to present a review lesson to their class on sketching the graph of a parabola in the xy-coordinate plane. They will decide to use the quadratic function f(x)=4x^2+8x-5 in the presentation. Each student will use algebra to explain how to find one of three key features of the graph. Angela writes the equation in factored form. Benjamin rewrites the equations by completing the square. Carla evaluates f(0). a. Sketch the graph of the function on the xy-coordinate grid. b. Describe how each student's work contributes to finding the key features of the graph. Complete their work and describe the key feature that is revealed. Write your descriptions and your work in the space provided.	○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connection

Science Connection (HS-PS2-1)

- A high school science class is conducting an experiment involving different methods of launching bottle rockets straight up into the air. They build the rockets and launch them one by one off of the roof of a 10-story building. The equation that models this type of projectile motion on Earth is $h(t)=-16t^2+v_0+h_0$, where h(t) represents the height of the projectile after t seconds, v0 is the initial velocity, and h0 is the initial height. This formula gives the height of the projectile as a function of time.
    - ⇒ Part A. When the class launches the first rocket, it has an initial velocity of 75 feet per second. It hits the ground after 6 seconds. (1) Determine the height of the building from which the rocket was launched. (2) Write the equation in standard form that represents the height of the rocket as a function of time. (3) Explain what the x- and y-values of the vertex of the parabola represent in terms of this scenario.
    - ⇒ Part B. The class launches a second rocket that has an initial velocity of 60 feet per second. (4) Use the height of the building you found in Part A to find how long, in seconds, it takes for this rocket to hit the ground after it was launched.
    - ⇒ Part C. A third rocket is launched with an initial velocity of 72 feet per second. (5) Determine the maximum height, in feet, the rocket reaches. Round your answer to the nearest whole number, if necessary. (6) Determine after how long, in seconds, it reached this height.
    - ⇒ Part D. A fourth and final rocket is launched with an initial velocity of 80 feet per second. (7) Determine for what values of t the height of the rocket is greater than 162 feet. Round your answer to the nearest tenth, if necessary.

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.12.ED.5: Evaluate the effectiveness of a product or system based on factors that are related to its requirements, specifications, and constraints (e.g., safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, ergonomics).
    - ⇒ Example: Students use graphing calculators and graph paper to reveal the strengths and weaknesses of technology associated with solving simple systems of linear and quadratic equations in two variables.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.1.12.CFR.4: Demonstrate an understanding of the interrelationships among attitudes, assumptions, and patterns of behavior regarding money, saving, investing, and work across cultures.
  - ⇒ Example: Graphing Systems of Equations and Savings Goals - In this activity, students will be able to use linear equations to compare savings balances over time for different individuals and families.

NJ SEL Competencies

- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals.
- Responsible Decision-Making: Develop, implement, and model effective problem solving and critical thinking skills.

Climate Change

- Linear Systems: This lesson applies linear systems to determine whether it is more environmentally friendly and cost-effective to own an electric vehicle rather than a gas vehicle.

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about building functions in two variables, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Encourage Student Leadership*: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare linear, quadratic and exponential models by creating problems together and deciding if the problems fit the necessary criteria. This experience

will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Relate the idea of adding, subtracting and multiplying complex numbers to whole numbers.
- Explain the background of complex numbers and connect to real life by explaining how they are used in electrical circuits.
- Use the example of the cyclical nature of the ones digit in the powers of 3 and connect it to the cyclical nature of the powers of i.
- Develop a reference document with students with verbal and pictorial descriptions.
- Model the thinking and procedure involved in solving a quadratic equation with complex solutions.
- Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence.
- Encourage students to verbalize their thinking while solving quadratic equations by asking, assessing and advancing questions.
- Link the concept of solving a system of equations

ELL:

- Describe and explain orally and in writing how to use properties of operations to add, subtract, and multiply complex numbers in the student's native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution.
- Encourage students to highlight like terms - use one color for the real parts and another color for the imaginary parts.
- Make a table with the words: Imaginary Unit, Complex Number and Imaginary Number, then write an example for each word in the column.
- Describe and explain how to solve quadratic equations with real coefficients that have complex solutions using in the student's native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution.
- Use a Venn diagram to represent the solution of a quadratic equation that expresses the relationship between Complex numbers and real

with one linear and one quadratic equation to solving a system of linear equations. ○ Model the thinking and processes necessary to decide on a solution path and solve a system with one linear equation and one quadratic equation accurately. ○ Provide students with reference sheets/notes to encourage confidence and independence. ○ Model the thinking and processes necessary to decide on a solution path and solve a system of three linear equations accurately. Provide students with reference sheets/notes to encourage confidence and independence.	numbers. ○ Let students discuss the possible form of the solution of quadratic equations. Indicate if students can obtain a real solution or an imaginary solution. ○ Demonstrate understanding of solving systems of linear and quadratic equations; then explain orally how to solve the equations in two variables in the student's native language and/or use gestures, equations and selected, technical words. ○ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty. ○ Use a graphic calculator to solve a linear system of equations to help students understand what various types of solutions might look like. ○ Provide students with construction paper, scissors and tape to build models of linear systems that have one, infinitely many solutions and no solutions. ○ Provide students with a visual representation of a linear system of three equations that students can recognize where the three planes intercept. ○ Let students verbalize the process of solving a system of three variables, in contract with the process of solving a two variables system.

Enrichment

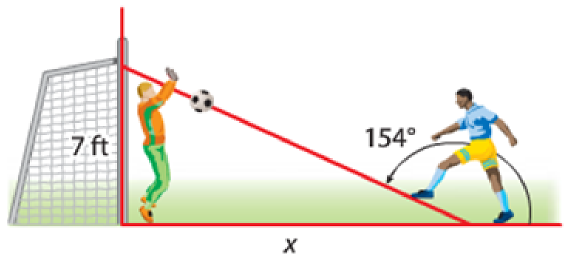
- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can extend their knowledge of special functions to graphing greatest integer functions.

- ⇒ Students can extend their knowledge of graphing quadratic inequalities to graphing absolute value inequalities.
- ⇒ Students can extend their knowledge of solving systems of equations in 3 variables to solving homogeneous systems.
- ⇒ Students can extend their knowledge of complex numbers to conjugates and absolute value.
- ⇒ Students can extend their knowledge of solving quadratic equations to writing quadratic equations given roots using the sum and product of roots.

Unit 2: Trigonometry	
Desired Outcomes	
NJSLS Mathematics 🟩 Major Content 🔷 Supporting Content 🟡 Additional Content + College and Career Readiness *Functions* ● Trigonometric Functions (F-TF) 🟡 Extend the domain of trigonometric functions using the unit circle (F-TF.1). 🟡 Model periodic phenomena with trigonometric functions (F.TF.5). 🟡 Prove and apply trigonometric identities (F.TF.8). **NJSLS Mathematical Practices** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
Enduring Understandings: ○ algebraic and geometric properties are used in trigonometry to solve problems and justify	**Essential Questions:** ○ What factors can be used to determine whether an analytic or graphical strategy is most

reasoning.	advantageous in solving a problem? ○ How are the properties of algebra and geometry used to obtain the trigonometric functions values? ○ How are the properties of algebra and geometry used to obtain the trigonometric functions values?
Students will know: ○ inverse trigonometric functions can be used to find one solution; using periodicity can be used to find all solutions. ○ the inverses of trigonometric functions can be used to solve for unknown information in right triangles. ○ the relationship between the three basic trig functions and how one can be used to find the others. ○ standard position of an angle includes the initial side on the positive x-axis angles can be sketched in degree or radian measure ○ the Unit Circle is a circle of radius 1, with 16 special angles in both degree and radian measurement, with a coinciding coordinate pair at each angle. ○ SOH-CAH-TOA can be extended to apply to the Unit Circle	**Students will be able to:** ○ find values of trigonometric functions for acute angles. ○ use trigonometric functions to find side lengths and angle measures of right triangles. ○ solve for all unknown parts of a right triangle by implementing properties of the trigonometric functions and their inverses. ○ draw and find angles in standard position. ○ convert between degree measures and radian measures. ○ find values of trigonometric functions based on the unit circle. ○ use the properties of periodic functions to evaluate trigonometric functions.
Assessment Evidence	
Suggested Performance Tasks:	**Required District/State Assessments:**

- NCTM “Angle of Elevation” and “Angle of Depression”
- A soccer player x feet from the goalie kicks the ball toward the goal, as shown in the figure. The goalie jumps up and catches the ball 7 feet in the air.
    a. Find the reference angle. then write a trigonometric function that can be used to find how far from the goalie the soccer player was when he kicked the ball.
    b. About how far away from the goalie was the soccer player?



- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes

- In class activities (proofs matching, scavenger hunts, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connection (HS-PS4-1):

- Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.
  - ⇒ Example: Provide an opportunity for students to visualize what sine, cosine, and tangent look like graphically using the visible light spectrum diagram.

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task
  - ⇒ Example: Students use graphing calculators to calculate the reference angle or the sides of a right triangle associated with trigonometric functions.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
  - ⇒ Example: Students will be given a real-world application problem involving trigonometric functions that will allow them to rationalize to their peers which trigonometric ratio should be applied.
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
  - ⇒ Example: Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as graphing calculators and technology to deepen their understanding of solving trigonometric functions.

NJ SEL Competencies

- Social Awareness: Demonstrate an awareness of the expectations for social interactions in a variety of settings
- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures.
    - ⇒ Example: When learning about trigonometric functions, students can create a real-world problem related to their interests or career major.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued.
    - ⇒ Example: Students will be provided with poster boards and have the opportunity to present a real-world problem of their creation to the rest of the class. This will allow everyone's voice to be heard, as well as allow students to learn more about their peers.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Extend the domain of trigonometric functions using the unit circle in order to relate the idea of degrees to radians.
- Provide students with the use of graphing calculators and technology where appropriate to visualize trigonometric functions.
- Explain the background of trigonometric functions and connect to real life by explaining how it is used to find the height of an object.
- Describe the relationship of how the Pythagorean

ELL:

- Describe and explain orally and in writing how to use trigonometric functions in the student's native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution.
- Let students discuss the possible ways of labeling a right triangle based on the reference angle that will relate writing the six trigonometric functions.
- Demonstrate understanding of solving trigonometric functions; then explain orally how to solve those functions in the student's native

Theorem relates to trigonometric functions. ○ Use relevant vocabulary, notations and symbols when appropriate. (Example: The difference between the trigonometric ratios.) ○ Provide students with a graphic organizer for the Unit Circle.	language and/or use gestures, equations and selected, technical words. ○ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty. ○ Use a graphing calculator to solve trigonometric functions to help students understand what various types of solutions might look like. (Example: Students should understand when angle measures should be in degrees or sides in radians.) ○ Provide students with a visual representation of what each trigonometric ratio represents in a right triangle. ○ Let students demonstrate the process of modeling application problems involving trigonometric functions.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students will extend their domain of trigonometric functions by creating the Unit Circle.
  - ⇒ Students can graph sine, cosine and tangent and explain transformations of these graphs.

Unit 3: Exponential and Logarithmic Functions

Desired Outcomes

<u>NJSLS Mathematics</u>

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Algebra

- Seeing Structure in Expressions (A-SSE)
  - 🟩 Interpret the structure of expressions (A-SSE.2).
  - 🟩 Write expressions in equivalent forms to solve problems (A-SSE.3c).
- Creating Equations (A-CED)
  - 🔷 Create equations that describe numbers or relationships (A-CED.1).

Functions

- Interpreting Functions (F-IF)
  - 🔷 Analyze functions using different representations (F-IF.7.e, F-IF.8b).
- Building Functions (F-BF)
  - 🟡 Build new functions from existing functions (F-BF.3).
- Linear, Quadratic, and Exponential (F-LE)
  - 🔷 Construct and compare linear, quadratic, and exponential models and solve problems (F-LE.3, F-LE.4, F-LE.5).

<u>NJSLS Mathematical Practices</u>

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Graphs provide a means of seeing correlation of a function. ○ Graphs can be examined in detail or in their entirety depending on what ○ information is sought. ○ Inverses of functions provide information to solve problems. ○ The characteristics of exponential and logarithmic functions and their representations are useful when solving real world problems.	○ How do exponential and logarithmic functions model real world problems and their solutions? ○ What do the asymptotes tell about the exponential and logarithmic functions? ○ What is the correlation between the graphical and algebraic representations of a logarithmic function? ○ How are exponents and logarithms related?
Students will know:	**Students will be able to:**
○ key features of the graphs of exponential and logarithmic functions (domain, range, intercepts, increasing/decreasing intervals, asymptotes). ○ transformations for exponential and logarithmic functions. ○ logarithmic functions are the inverse of the exponential functions. ○ exponential rules are used to simplify exponential and logarithmic expressions. ○ how to rewrite exponential and logarithmic expressions using exponent rules. ○ evaluate exponential and logarithmic expressions	○ Graph exponential functions and apply transformations to exponential functions. ○ Solve exponential equations with applications. ○ Analyze a situation modeled by an exponential function, such exponential growth, exponential decay and compounding interest. ○ Develop the definition of logarithms by exploring and describing the relationships between exponential functions and their inverses. ○ Graph logarithmic functions. ○ Solve logarithmic equations by applying properties of logarithms.

and equations. ○ problems can be modeled using exponential or logarithmic functions.	○ Solve application problems involving logarithmic equations. ○ Determine solutions of exponential and logarithmic equations and inequalities using graphs, tables and algebraic methods. Interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities. ○ Describe limitations on domains and ranges, and examine asymptotic behavior. ○ Use parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions. ○ Use relevant vocabulary, notations, and symbols when appropriate. ○ Use graphing calculators and technology where appropriate.
Assessment Evidence	
Suggested Performance Tasks: ○ CITY COUNCIL: An analyst studying the population of a town determines that the population can be modeled by the formula f(t)=120,000(1.015)^t, where f(t) represents the population after t years. A city council member makes this claim: “Based on the formula, after 1 year the population will have increased by 1,800. Since 1,800 divided by 12 is 150, we can use the fact that the population increases by 150 people per month to predict the future population of the town.” Explain why the city council member’s claim is or is not a valid way to predict the	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct)

population. Then, modify the initial formula such that it represents the predicted population after m months, and use the modified formula to predict the population after 50 months. ○ REASONING: A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the pattern is 18. a. Write the function that represents the situation. b. Classify the function as either exponential growth or decay, and identify the growth or decay factor. Then graph the function for the first 10 numbers. c. What is the value of the tenth number? Round to the nearest whole number. ○ ARGUMENTS: Consider y=log_bx in which b, x, and y are real numbers. Zero can be in the domain sometimes, always, or never. Justify your answer. ○ PRECISION: Five years ago the grizzly bear population in a certain national park was 325. Today it is 450. Studies show that the park can support a population of 750. a. What is the average annual rate of growth in the population if the grizzly bears reproduce once a year? b. How many years will it take to reach the maximum population if the population growth continues at the same average rate?	○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework
Learning Plan	

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connection

- HS-LS1-4: Growth and Development of Organisms: In multicellular organisms individual cells grow and then divide via a process called mitosis, thereby allowing the organism to grow. The organism begins as a single cell (fertilized egg) that divides successively to produce many cells, with each parent cell passing identical genetic material (two variants of each chromosome pair) to both daughter cells. Cellular division and differentiation produce and maintain a complex organism, composed of systems of tissues and organs that work together to meet the needs of the whole organism. (LS1.B)
  - ⇒ Example: A type of bacteria is growing exponentially according to the model y=1000e^kt, where t is the time in minutes. (See Teacher Edition pg 511)
    - Part A. If there are 1000 cells initially and 1650 cells after 40 minutes, find the value of k for the bacteria.
    - Part B. Suppose a second type of bacteria is growing exponentially according to the model $y=50e^{0.0432t}$. Determine how long it will be before the number of cells of this bacteria exceed the number of cells in the other bacteria.
- HS-PS1-8: Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay
  - ⇒ Example: The half-life of Sodium-22 is 2.6 years. Determine the value of k and the equation of the k for Sodium-22. (See Teacher Edition pg 510)

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
  - ⇒ Example: Students use graphing calculators and graph paper to reveal the strengths and weaknesses of technology associated with solving exponential and logarithmic functions.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.1.12.FP.3: Relate the concept of delayed gratification (i.e., psychological distance) to meeting financial goals, investing and building wealth over time.
  - ⇒ Example: Exponential Regression and Investing Trends - In this activity, students will be able to analyze data on the growth of robo-advisors and compare the performance of two funds.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
  - ⇒ Example: Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments. They will be able to explain how to solve exponential and logarithmic functions.

NJ SEL Competencies

- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges
- Self-Management: Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals

Climate Change

- Generating Heat: Students graph carbon emissions and population growth over time and comment on the trends they observe. [Resource Location: Math Shared Drive > Interdisciplinary Connections > Climate Change > Generating Heat]

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student

interests and cultures.
    - ⇒ Example: When learning about writing equations for exponential decay, students will be placed into cooperative groups to carry out an activity using M&Ms to model exponential growth and decay.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued.
    - ⇒ Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Encourage Student Leadership*: Create an avenue for students to propose problem solving strategies and potential projects.
    - ⇒ Example: Students can work in groups to research a topic that relates to exponential growth, exponential decay, and logarithmic functions. Students can then create a scenario applying the concept they have researched. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms.
    - ⇒ Example: Teach math vocabulary in various modalities for students to remember. Use multi-model activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:	ELL:
○ Relate the ideas of transformations of functions to the graph of an exponential function. ○ Provide students with the use of graphing calculators and technology where appropriate to visualize exponential growth and decay. ○ Explain the background of exponential functions	○ Describe and explain orally and in writing how to use properties of exponential and logarithmic equations in the student's native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution. ○ Encourage students to highlight like terms - use

and connect to real life by explaining how they are used in determining blood toxicity levels. ◦ Describe the relationship between solving exponential equations and inequalities. ◦ Use relevant vocabulary, notations and symbols when appropriate. (Example: The difference between equations and inequalities, as well as the differences between bases of logarithms) ◦ Provide students with a graphic organizer for applying properties of logarithmic functions.	one color for bases and a different color for exponents. ◦ Make a table with the words: Exponential Growth, Exponential Decay, Logarithmic Equation, Logarithmic Inequalities, and Properties of Logarithms then write an example for each word in the column. ◦ Let students discuss the possible forms of the solution of exponential growth, exponential decay and logarithmic functions. ◦ Demonstrate understanding of solving exponential equations and inequalities, exponential transformations, logarithmic equations and inequalities, and applying the properties; then explain orally how to solve those functions in the student's native language and/or use gestures, equations and selected, technical words. ◦ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty. ◦ Use a graphing calculator to solve exponential equations and logarithmic functions to help students understand what various types of solutions might look like. ◦ Provide students with a visual representation of exponential equations that students can recognize where "a" is the initial value and "b" is the rate of growth or decay. ◦ Let students demonstrate the process of changing the base of logarithmic equations and inequalities.

Enrichment
○ Challenge problems from resource sets ○ Extended learning goals: ⇒ Students can extend their knowledge of logarithmic functions to logistical growth. ⇒ Students can analyze functions using different representations. ⇒ Students can construct and compare: linear, quadratic, exponential and logarithmic models to solve problems.

Unit 4: Higher Level Functions
Desired Outcomes

<u>NJSLS Mathematics</u>
 	
🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness
 	
Number and Quantity
• The Real Number System (N-RN)
    🟩 Extend the properties of exponents to rational exponents (N.RN.1. N.RN.2).
 	
Algebra
• Seeing Structure in Expressions (A-SSE)
    🟩 Interpret the structure of expressions (A-SSE.2).
• Arithmetic with Polynomials and Rational Expressions (A-APR)
    🟡 Perform arithmetic operations on polynomials (A-APR.1).
    🟩 Understand the relationship between zeros and factors of polynomials (A-APR.2, A-APR.3).
    🔷 Rewrite rational expressions (A-APR.6).
• Creating Equations (A-CED)
    🔷 Create equations that describe numbers or relationships (A-CED.1, A-CED.2).
• Reasoning with Equations and Inequalities (A-REI)
    🟩 Understand solving equations as a process of reasoning and explain the reasoning (A-REI.2).
 	
Functions
• Interpreting Functions (F-IF)
    🟩 Interpret functions that arise in applications in terms of the context (F-IF.4).
    🔷 Analyze functions using different representations (F-IF.7b, F-IF.7c, F-IF.8, F-IF.9).
• Building Functions (F-BF)

🟩 Build a function that models a relationship between two quantities (F-BF.1b).
🟡 Build new functions from existing functions (F-BF.3, F-BF.4a).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ the characteristics of polynomial functions and their representations are useful when solving real world problems. ○ a simplified version of an expression may be more useful ○ the number and type of solutions vary predictably based on the type of equation ○ interpretation of a solution is necessary in terms of its context ○ radical expressions can be simplified ○ radical equations can be solved by manipulation ○ changing the exponent of a function affects the domain and the number of solutions for that function.	○ What is the correlation between the graphical and algebraic representations of a polynomial function? ○ Why are domain restrictions necessary for rational functions? ○ What process is used to solve a rational equation? ○ How do arithmetic properties apply to radical expressions or other expressions with rational exponents? ○ How are arithmetic properties used to solve radical equations? ○ What effect does changing the exponent of a function have on the domain and the number of solutions for that function?
Students will know:	**Students will be able to:**

- the leading coefficient indicates the end behavior of a polynomial function.
- the maximum number of real zeros of a function coincide with its degree and are the x-intercepts of its graph.
- imaginary zeros are not indicated on the graph of a polynomial function.
- how to solve various polynomial equations by factoring.
- how to recognize the relationship between a zero and a factor.
- the properties of solving polynomial functions can be extended to include rational functions.
- restrictions on a rational function coincide with the extraneous solutions.
- radical expressions can be simplified through addition, subtraction, multiplication and division.
- radical expressions can be simplified using the properties of exponents.
- radical equations can be solved by using powers.

- Multiply, divide, and simplify monomials and expressions involving powers.
- Add, subtract, multiply, and divide (long division and synthetic division), polynomials.
- Evaluate polynomial functions. Identify general shapes of graphs of polynomials.
- Graph polynomial functions and locate their zeros. Find the relative maxima and minima of polynomial functions.
- Factor and solve polynomial equations.
- Evaluate functions by using synthetic substitution.
- Determine whether a binomial is a factor of a polynomial by using synthetic substitution.
- Determine the number and type of roots for a polynomial equation. Find the zeros of a polynomial function.
- Find the sum, difference, product, and quotient of functions.
- Find the inverse of a function or relation. Determine whether two functions or relations are inverses.
- Determine compositions of functions. Use compositions to draw connections between inverses.
- Graph and analyze square root functions and square root inequalities.
- Identify and sketch graphs of the square root parent function. Use the parent functions to investigate, describe and predict the effects of changes in a, h, and k in vertex form.
- Determine solutions of square root equations and inequalities using graphs, tables, and algebraic methods.

	○ Simplify radicals and radical expressions. ○ Add, subtract, multiply, and divide radical expressions. ○ Write expressions with rational exponents in radical form and vice versa. ○ Simplify expressions in exponential or radical form. ○ Solve equations and inequalities containing radicals. ○ Determine properties and graph transformations of reciprocal functions. ○ Graph rational functions with point discontinuity and vertical, horizontal, and oblique asymptotes. ○ Solve rational equations and inequalities.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Example 1: Diane owns a store that sells computers. Her profit, in dollars, is represented by the function (x)=x3 -22x2 -240x , where x is the number of computers sold. Diane hopes to make a profit of at least $10,000 by the time she sells 36 computers. a. Explain whether or not Diane will meet her goal. Justify your reasoning. b. Diane states that there are three possible values of x for which she will have a profit of $0. Find the values of x that produce a zero profit to show whether Diane is correct or not. Justify your reasoning. ○ Example 2: The time T in seconds that it takes a pendulum to make a complete swing back and	**<u>Required District/State Assessments:</u>** ○ Unit Assessment ○ SGO Assessments **<u>Suggested Formative/Summative Assessments:</u>** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes

forth is given by the formula $T=2\pi\sqrt{(L/g)}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches long. How long does it take for the pendulum to make a complete swing? b. A clockmaker wants to build a pendulum to make a complete swing back and forth. How long should the pendulum be? ○ Example 3: Modeling - According to the A.C. Nielsen Company, the average American watches 4 hours of television a day. a. Write an equation to represent the average number of hours spent watching television by household members during a period of d days. b. Assume that members of your household watch the same amount of television each day as the average American. How many hours of television would the members of your household watch in a week?	○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Unit pacing:

- Polynomials and Polynomial functions (14 days)
- Radical functions (14 days)
- Rational functions (7 days)

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connections:

- HS-ESS1-1. Develop a model based on evidence to illustrate the life span of the sun and the role of nuclear fusion in the sun's core to release energy that eventually reaches Earth in the form of radiation.
  - ⇒ Example: Students can use information about the amount of time it takes light to travel to Earth to make predictions about how long it would take the sun's light to reach other planets.
- HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.
  - ⇒ Example: Students can use the formula for voltage to determine the voltage if the power and current are represented by polynomials.
- HS-ESS1-4. Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.
  - ⇒ Example: Students can use Kepler's Laws of orbital motion to determine how far planets are from the sun.

Health and Science Career:

- 9.3.HL-DIA.5 Select, demonstrate and interpret diagnostic procedures.
  - ⇒ Example: Students can use the formula for volume of air in the lungs to determine an amount of air in the lungs and interpret its meaning in terms of a respiratory cycle.

Business Management and Administration:

- 9.3.12.BM.1 Utilize mathematical concepts, skills and problem solving to obtain necessary information for

decision-making in business.
    ⇒ Example: Students can use profit models to determine when a business will make or lose money.

Hospitality and Tourism Career:

- 9.3.HT-TT.2 Apply unit and time conversion skills to develop travel schedules and compute cost, distance and time (including travel time) factors.
    - ⇒ Example: Students can use inverses to convert between units of money (ie dollars to pesos).

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
    - ⇒ Example: Students use graphing calculators and graph paper to reveal the strengths and weaknesses of technology associated with solving higher level functions.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
    - ⇒ Example: Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as graphing calculators and technology to deepen their understanding of solving higher level functions.

NJ SEL Competencies

- Relationship skills: Identify who, when, where, or how to seek help for oneself or others when needed.
- Responsible Decision-Making: Develop, implement, and model effective problem-solving and critical thinking skills.

Culturally Relevant Connections

- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- *Encourage Student Leadership*: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can explore properties of circles and explain their findings in small groups.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Review properties of exponents by using examples, having students write them out as multiplication or division, and simplify.
- Create anchor charts of how to simplify monomials, simplify radical expressions, and determine asymptotes and display them in the classroom for student reference.
- Use reference sheets that list step-by-step procedures and models strategies for synthetic division, composition of functions, transformations of square root functions and reciprocal functions, rationalizing denominators, solving radical equations and inequalities, and solving rational equations and inequalities.
- Create a guided note sheet for the end behavior of common polynomial functions and have students fill in information as each function is investigated in class.

ELL:

- Describe and explain orally and in writing how to use properties of exponents, operations on functions, and interpret expressions and equations that represents a quantity in terms of its context (rational and reciprocal functions) in the student's native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution.
- Let students discuss the possible forms of the solution of solving operations of polynomial expressions and equations.
- Demonstrate understanding of properties of exponents, operations on functions, and interpret expressions and equations that represents a quantity in terms of its context (rational and reciprocal functions), and applying the properties; then explain orally how to solve those functions in the student's native language and/or use gestures, equations and selected, technical words.

○ Create an anchor chart for factoring techniques of polynomials with examples for student reference. ○ Provide students with the use of graphing calculators and technology where appropriate to visualize square root and reciprocal functions and end behaviors.	○ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty. ○ Use a graphing calculator to graph the rational and reciprocal functions to help students understand what various types of solutions might look like. ○ Provide students with a visual representation of asymptotes (vertical, horizontal, slant and point of discontinuity). ○ Let students demonstrate the process of dividing polynomials (long and synthetic division) and composition of functions.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can perform dimensional analysis while calculating polynomial sums, differences, products, and quotients (5.1 Algebra Lab: Dimensional Analysis).
  - ⇒ Students can use a graphing calculator to divide polynomials (5.2 Graphing Technology Lab: Dividing Polynomials).
  - ⇒ Students can graph power functions and use inductive reasoning to determine transformations from the parent function (5.3 Graphing Technology Lab: Power Functions).
  - ⇒ Students can determine polynomial functions of best fit for data (5.3 Enrichment: Approximation by Means of Polynomials and 5.4 Graphing Technology Lab: Modeling Data Using Polynomial Functions).
  - ⇒ Students can use a graphing calculator to solve polynomial equations (5.5 Graphing Technology Lab: Solving Polynomial Equations by Graphing)
  - ⇒ Students can use technology (such as a spreadsheet) to determine whether a polynomial equation is an identity (5.5 Graphing Technology Lab: Polynomial Identities).
  - ⇒ Students can apply the bisection method to approximate zeros of a polynomial function (5.7 Enrichment: The Bisection Method for Approximating Real Zeros)

- ⇒ Students can use a graphing calculator to identify polynomial characteristics, such as real zeros, maximum and minimum points, numbers and types of zeros, y-intercepts, and symmetry (5.7 Graphing Technology Lab: Analyzing Polynomial Functions).
- ⇒ Students can use graphing calculators to approximate relative minima and maxima of polynomial functions (6.1 Enrichment: Relative Maximum Values).
- ⇒ Students can use a graphing calculator to compare functions and their inverses using tables and graphs (6.2 Graphing Technology Lab: Inverse Functions and Relations)
- ⇒ Students can use a graphing calculator to graph nth root functions (6.4 Graphing Technology Lab: Graphing nth Root Functions)
- ⇒ Students can use the zero feature on their graphing calculator to solve radical equations and inequalities (6.7 Graphing Technology Lab: Solving Radical Equations and Inequalities).
- ⇒ Students can use sign charts to determine the characteristics of graphs for rational functions (8.4 Enrichment: Characteristics of Rational Function Graphs)
- ⇒ Students can use graphing calculators to explore graphs of rational functions (8.4 Graphing Technology Lab: Graphing Rational Functions)
- ⇒ Students can determine planes of discontinuity and asymptotes for three dimensional curves (8.6 Enrichment: Asymptotes in Three-Dimensions).
- ⇒ Students can use a graphing calculator to solve rational equations by graphing or using a table (8.6 Graphing Technology Lab: Solving Rational Equations and Inequalities)

Unit 5: Statistical Analysis	
Desired Outcomes	

NJSLS Mathematics
 	
🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness
 	
Statistics and Probability
• Making Inferences and Justifying Conclusions (S-IC)
    🔷 Understand and evaluate random processes underlying statistical experiments (S-IC.1).
    🟩 Make inferences and justify conclusions from sample surveys, experiments and observational studies (S.IC.4).
• Interpreting Categorical and Quantitative Data (S-ID)
    🟡 Summarize, represent, and interpret data on a single count or measurement variable (S-ID.4).
 	
NJSLS Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.	

Enduring Understandings:	**Essential Questions:**

○ the shape of a distribution is used to select appropriate statistics and compare data. ○ the standard normal curve allows for the comparison of data from different normal distributions. ○ the standard deviation and the mean affect the graph of the normal distribution.	○ How well can we predict the outcomes of future events? ○ What are possible outcomes of situations? ○ How can critical vocabulary be used to better enhance the communication and understanding of mathematics?
<u>Students will know:</u> ○ vocabulary associated with probability and statistics. ○ a distribution of data shows the frequency of each possible data value. ○ the shape of a distribution can be determined by looking at its histogram or box-and-whisker plot. ○ Areas under the curve represent probabilities associated with continuous distributions. ○ The normal curve is a probability distribution and the total area under the curve is 1. ○ For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. ○ The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. ○ A z-score expresses, in standard deviation units, how far an element falls from the mean of the data set.	**<u>Students will be able to:</u>** ○ Use shapes of distributions to select appropriate statistics. ○ Use the shapes of distributions to compare data. ○ Use the Empirical Rule to analyze normally distributed variables. ○ Use a measure of central tendency to represent a set of data and find measures of variation for a set of data. ○ Apply the standard normal distribution and z-values. ○ Create and use graphs of the Normal Distribution and solve problems using these graphs.

Assessment Evidence	
Suggested Performance Tasks: ○ The weight of a bag of Brand A cookies is labeled as 4 ounces on the bag. However, the actual weights of the bags vary by a small amount. According to the packaging specifications, the weights are approximately normally distributed with a mean of 4.10 ounces and a standard deviation of 0.10 ounces. a. According to the specifications, find the approximate percent of the bags that weigh 4.00 ounces or more. b. During a quality control check on the bag weights, a bag was found that weighed 3.95 ounces. How many standard deviations below the mean was the bag weight?	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan
Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Language Arts (RI.11-12.7):

- Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
  - ⇒ Example: Students can complete "Describing and Comparing Data Distributions (Census Activity)" from the Shared Drive.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
  - ⇒ Example: Students will make sound judgments about the use of specific tools (i.e. graphing calculators, Desmos, Google Sheets, etc.) to deepen their understanding of analyzing distributions of data.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
  - ⇒ Example: In this activity, students will be able to calculate mean, median, and mode; analyze data on

college costs.

- 9.1.12.PB.6: Describe and calculate interest and fees that are applied to various forms of spending, debt and saving
  - ⇒ Example: In this activity, students will be able to plot sample banking fees using a box-and-whisker plot; analyze the interquartile ranges of a data set.

NJ SEL Competencies

- Relationship Skills: Identify who, when, where, or how to seek help for oneself or others when needed
- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures.
  - ⇒ Example: When learning about distributions of data, students will have to analyze and interpret data.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued.
  - ⇒ Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Provide students with the use of graphing calculators and technology where appropriate to visualize the shape of a distribution for probability

ELL:

- Describe and explain orally and in writing how to use properties of distributions of data and the Empirical Rule in the student's native language and/or use selected technical vocabulary in

and statistics. ○ Represent and interpret data of probability distributions. (Example: Skewed vs symmetric) ○ Describe the relationship between the expected value and standard deviation of distribution data. ○ Use relevant vocabulary, notations and symbols when appropriate. (Example: If the data distribution is discrete or continuous.) ○ Provide students with a graphic organizer for the Empirical Rule.	phrases and short sentences with equations to explain the solution. ○ Encourage students to highlight the different sections of the Empirical Rule- use a different color for each of the Standard Deviations 1, 2, and 3, as well as the area under the curve that corresponds to each part. ○ Make a table with the words: negatively skewed, symmetric, and positively skewed; then write an example for each word in the column. ○ Let students discuss the possible forms of data distributions and how to compare the distributions. ○ Demonstrate understanding of solving distributions of data, probability distribution and the Empirical Rule; then explain orally how to solve those functions in the student's native language and/or use gestures, equations and selected, technical words. ○ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty. ○ Use a graphing calculator to solve distributions of data, probability distributions, and the Empirical Rule to help students understand what various types of solutions might look like. ○ Let students demonstrate the process analyzing distributions of data (mean and standard deviation, five number summary, expected value, and discrete or continuous).

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can extend their knowledge of distributions of data by understanding and evaluating random processes underlying statistical experiments.
  - ⇒ Students can analyze and justify conclusions from samples, surveys, experiments, and observational data studies.
  - ⇒ Students can construct and compare: histograms, box-and-whisker plots, and normal distribution curves of data.

Appendix A: Culturally Relevant Pedagogy Examples

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION	KNOWLEDGE CONSTRUCTION	PREJUDICE REDUCTION	EQUITABLE PEDAGOGY	EMPOWERING SCHOOL CULTURE
Teachers use examples and content from a variety of cultures & groups.	Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.	Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.	Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.	Using the other four dimensions to create a safe and healthy educational environment for all.
This unit / lesson is connected to other topics explored with students. There are multiple viewpoints reflected in the content of this unit / lesson. The materials and resources are reflective of the diverse identities and experiences of students. The content affirms students, as well as exposes them to experiences other than their own.	This unit / lesson provides context to the history of privilege and oppression. This unit / lesson addresses power relationships. This unit / lesson help students to develop research and critical thinking skills. This curriculum creates windows and mirrors* for students.	This unit / lesson help students question and unpack biases & stereotypes. This unit / lesson help students examine, research and question information and sources. The curriculum encourage discussion and understanding about the groups of people being represented. This unit / lesson challenges dominant perspectives.	The instruction has been modified to meet the needs of each student. Students feel respected and their cultural identities are valued. Additional supports have been provided for students to become successful and independent learners. Opportunities are provided for student to reflect on their learning and provide feedback.	There are opportunities for students to connect with the community. My classroom is welcoming and supportive for all students? I am aware of and sensitive to the needs of my students and their families. There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition). New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

Interactive Supports*

- In a whole group
- In a small group
- With a partner such as *Turn-and-Talk*
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as *Think-Pair-Share*
- Interactive websites or software
- With a mentor or coach

Verbal and Textual Supports

- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as *What? When? Where? How? Why?*
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

Appendix C: WIDA ELD Standards Integration

ELD-MA.9-12 Explain Interpretive	Interpret mathematical explanations by • Identifying concept or entity • Analyzing data and owning problem-solving approaches • Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles
ELD-MA 9-12 Explain Expressive	Construct mathematical explanations that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions
ELD-MA.9-12 Argue Interpretive	Interpret mathematics arguments by • Comparing conjectures with previously established results and stated assumptions • Distinguishing correct from flawed logic • Evaluating relationships among evidence and mathematical principles to create generalizations
ELD-MA.9-12 Argue Expressive	Construct mathematics arguments that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions

Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is ...	**Enrichment is not...**
• Planned and purposeful • Different, or differentiated, work – not just more work • Responsive to students’ needs and situations • A promotion of high-level thinking skills and making connections within content • The ability to apply different or multiple strategies to the content • The ability to synthesize concepts and make real world and cross curricular connections • Elevated contextual complexity • Sometimes independent activities, sometimes direct instruction • Inquiry based or open-ended assignments and projects • Using supplementary materials in addition to the normal range of resources • Choices for students • Tiered/Multi-level activities with flexible groups (may change daily or weekly)	• Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) • Worksheets that are more of the same (busywork) • Random assignments, games, or puzzles not connected to the content areas or areas of student interest • Extra homework • A package that is the same for everyone • Thinking skills taught in isolation • Unstructured free time

Appendix F: Resources

Textbook: Carter, John, et al, Algebra 2, Glencoe, McGraw-Hill, 2010 and 2012

POR Text: Bellman, Bragg, & Handlin, Algebra 2, Prentice Hall, Pearson, 2011

Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.